UNICUS OLYMPIADS

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Sample Paper

Class 10

Unicus Global Mathematics Olympiad (UGMO)

Time: 60 minutes

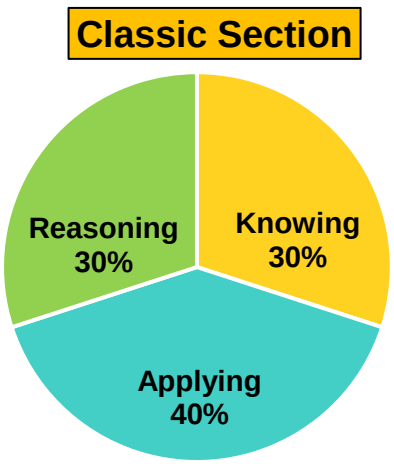
Pattern and Marking Scheme			
Section	**Total Questions**	**Marks per Question**	**Total Marks**
Classic Section	30	1	30
Scholar Section	15	2	30
Grand Total	**45**		60

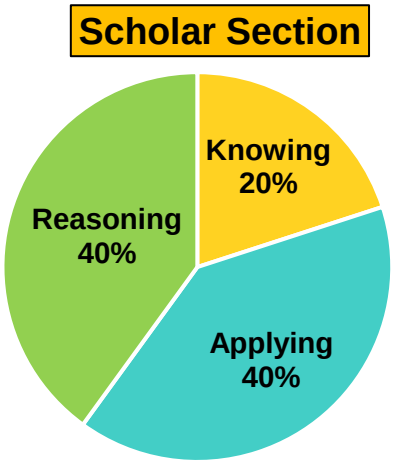
Unicus Global Mathematics Olympiad (UGMO)

The **Unicus Global Olympiad** is organised around two dimensions:

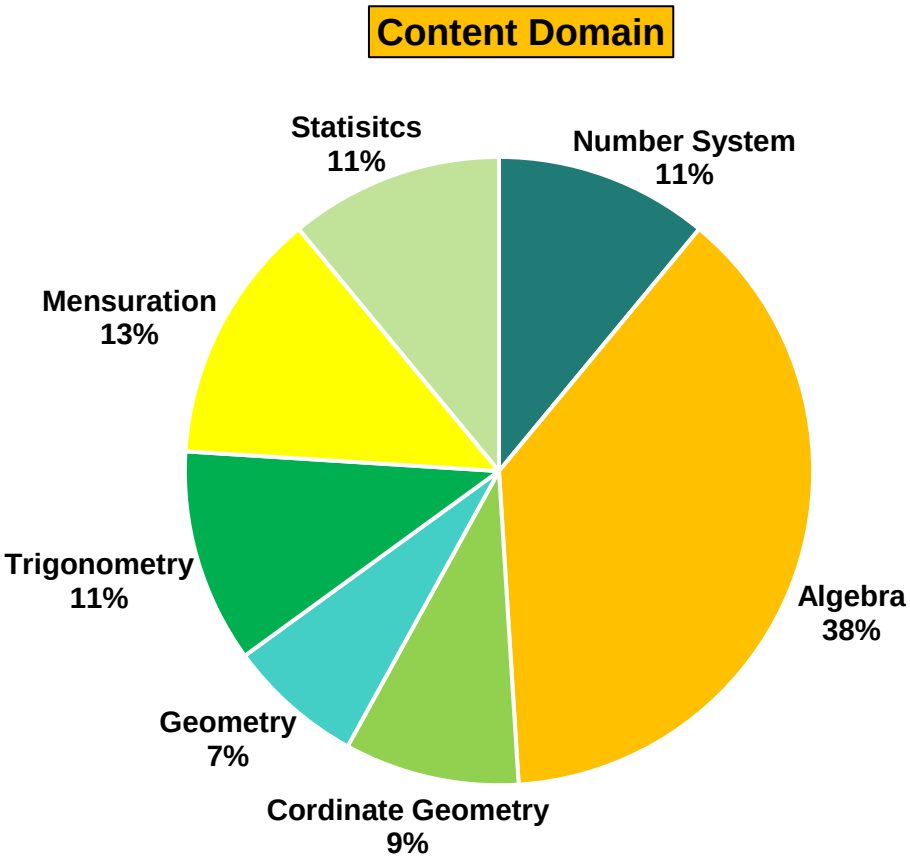
1. Content dimension, specifying the subject matter domains to be assessed
2. Cognitive dimension, specifying the thinking processes to be assessed

Target percentages of the question paper devoted to cognitive domains





Target percentages of the question paper devoted to content domains



For more details, visit https://www.unicusolympiads.com/.

Classic Section (Each Question is 1 Mark)

Cognitive Domain: Knowing	Content Domain: Number system

1. The sum of the L.C.M. and H.C.F. of two numbers is 1260. If their L.C.M. is 900 more than their H.C.F. Find the product of two numbers.

a. 194400
b. 203400
c. 198400
d. 205400

Cognitive Domain: Applying	Content Domain: Number system

2. If $x = (4 + \sqrt{15})^{1/3} + (4 - \sqrt{15})^{1/3}$, then find the value of $x^3 - 3x$.

a. 5
b. 8
c. 4
d. 12

Cognitive Domain: Applying	Content Domain: Number system

3. If $x = 2/(\sqrt{10} - \sqrt{8})$, $y = 2/(\sqrt{10} + 2\sqrt{2})$, then find $(x - y)^2$.

a. $8\sqrt{2}$
b. $4\sqrt{2}$
c. 32
d. 64

Cognitive Domain: Knowing	Content Domain: Number system

4. Solve for x : $\log(3 + 2\log(1 + x)) = 0$.

a. -9/10
b. 1/10
c. -1
d. 1/7

Cognitive Domain: Knowing	Content Domain: Algebra

5. If $x^3 + 2x^2 + ax + b$ has factors x + 1 and x - 1, find a and b.

a. 1, -2
b. -1, -2
c. -2, -1
d. 1, 2

Cognitive Domain: Reasoning	Content Domain: Algebra

6. The polynomial f(x) has roots of the equations 3, -3 and -k. Given that the coefficient of x^3 is 2 and that f(x) has a remainder of 8, when divided by x + 1, find the value of k.

a. -2
b. -1
c. 1
d. 0

Cognitive Domain: Applying	Content Domain: Algebra

7. Two pipes are used to fill a swimming pool in 12 hours. If the larger pipe is used for four hours and the smaller pipe for nine hours, only half of the pool is filled. How long would it take larger pipes and smaller pipes to fill the pool on their own?

a. 30 hours, 20 hours
b. 20 hours, 30 hours
c. 10 hours, 30 hours
d. 30 hours, 10 hours

Cognitive Domain: Knowing	Content Domain: Algebra

8. Find the solution set of $1/(2x - 4) < 0$.

a. $(-\infty, \infty)$
b. $[2, \infty)$
c. $(2, \infty)$
d. $(-\infty, 2]$

Cognitive Domain: Applying	Content Domain: Algebra

9. Find the two natural numbers so that their sum cannot exceed 6 and the difference between first and second number is positive and does not exceed 2 and also the resultant sum is maximum.

a. (4, 2)
b. (1, 3)
c. (2, 2)
d. (1, 5)

Cognitive Domain: Reasoning	Content Domain: Algebra

10. A classroom can fit at least 9 tables with an area of 1 m^2 and the perimeter of the classroom is 12 m. Find the bounds on the length and breadth of the classroom.

a. length < 1 m, breadth > 4 m
b. length < 3 m, breadth > 3 m
c. length < 5 m, breadth > 5 m
d. length < 2 m, breadth > 2 m

Cognitive Domain: Knowing	Content Domain: Algebra

11. Find the equation whose roots are the reciprocals of the roots of $3x^2 - 5x + 7 = 0$.

a. $2x^2 - 5x + 9 = 0$
b. $5x^2 - 5x + 7 = 0$
c. $7x^2 - 5x + 3 = 0$
d. $5x^2 - 7x + 3 = 0$

Cognitive Domain: Applying	Content Domain: Algebra

12. An aeroplane travelled a distance of 800 km at an average speed of x km/hr. On the return journey, the speed was increased by 80 km/hr. If the return journey took 30 minutes less than the onward journey, find the average aeroplane speed.

a. 160 km/h
b. 400 km/h
c. 420 km/h
d. 320 km/h

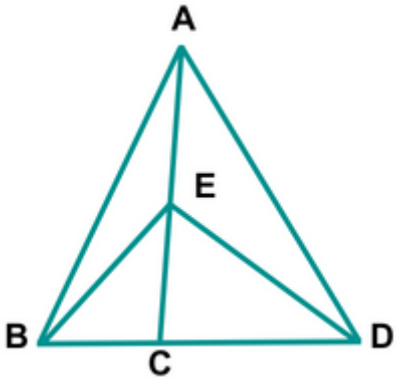
Cognitive Domain: Reasoning	Content Domain: Algebra

13. Out of a group of employees, twice the square root of the number of employees are on a trip to attend a conference held by the company, half the number are in the office while the remaining 6 are on leave. What is the number of employees in the group?

a. 36
b. 42
c. 54
d. 60

Cognitive Domain: Applying	Content Domain: Geometry

14. If BC : CD = 2 : 3, AE : EC = 3 : 4 and BC : AE = 2 : 3, then find the ratio of the area of ∆ECD to the area of ∆AEB.



a. 4 : 3
b. 2 : 3
c. 2 : 1
d. 3 : 5

Cognitive Domain: Knowing	Content Domain: Geometry

15. The perimeters of two similar triangles ABC and PQR are 72 cm and 48 cm, respectively. If PQ = 20 cm, then find the length of AB.

a. 60 cm
b. 30 cm
c. 15 cm
d. 24 cm

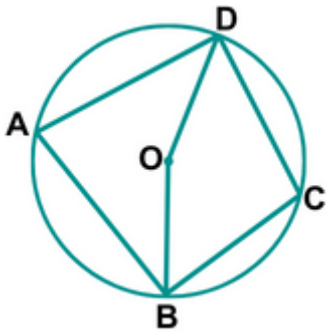
Cognitive Domain: Applying	Content Domain: Algebra

16. The sum of the first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the fourth term of the G.P.

a. 64/7
b. 16/7
c. 128/7
d. 256/7

Cognitive Domain: Applying	Content Domain: Geometry

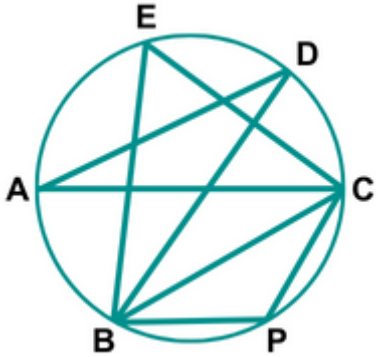
17. In the shown figure, O is the centre of the circle. BC and CD are equal chords. If ∠OBC = 55°, then find ∠BAD.



a. 70°
b. 60°
c. 80°
d. 45°

Cognitive Domain: Reasoning	Content Domain: Geometry

18. In the figure above (not to scale) AC is the diameter of the circle and ∠ADB = 20°, then find ∠BPC.



a. 80°
b. 110°
c. 90°
d. 100°

Cognitive Domain: Knowing	Content Domain: Trigonometry

19. Evaluate:

$$\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

a. cot θ - cosec θ
b. sec θ - cot θ
c. cosec θ - cot θ
d. tan θ - cot θ

Cognitive Domain: Applying	Content Domain: Trigonometry

20. A vertical pole consists of two parts, the lower part being one-third of the whole. At a point in the horizontal plane through the base of the pole and a distance of 20 m from it, the upper part of the pole subtends an angle whose tangent is 1/2. What are the possible heights of the pole?

a. 20 m or $20\sqrt{3}$ m
b. 20 m or 60 m
c. 16 m or 48 m
d. 40 m or 120 m

Cognitive Domain: Reasoning	Content Domain: Trigonometry

21. A spherical balloon of radius r subtends an angle α at the eye of an observer, while the angle of elevation of its centre is β. Find the height of the centre of the balloon.

a. r sin α/2. cos β
b. r sec α/2. sin β
c. r cosec α/2. sin β
d. r cos α/2 .sin β

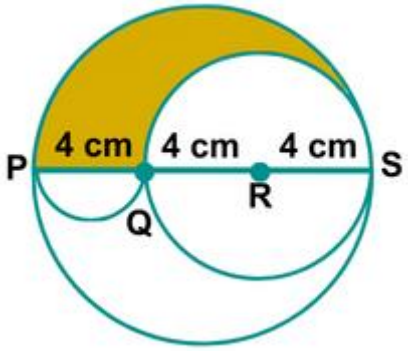
Cognitive Domain: Reasoning	Content Domain: Mensuration

22. A sector of a circle of radius 6 cm has an angle of 120°. It is rolled up so that the two bounding radii are joined together to form a cone. Find the total surface area of the cone and the volume of the cone.

a. 8 π cm^2, $(8\ \pi\ \sqrt{2})/3\ cm^3$
b. 24 π cm^2, $(24\sqrt{2}\ \pi)/3\ cm^3$
c. 32 π cm^2, $(32\sqrt{2}\ \pi\ )/3\ cm^3$
d. 16 π cm^2, $(16\sqrt{2}\ \pi)/3\ cm^3$

Cognitive Domain: Applying	Content Domain: Mensuration

23. A circle with a diameter PQRS, where the radius of the circle is 6 cm. The lengths PQ, QR, and RS are all equal. Semi-circles are drawn on PQ and QS as diameters. Determine the ratio of the area of the shaded region to the area of the unshaded region.



a. 5 : 13
b. 1 : 2
c. 25 : 121
d. 5 : 12

Cognitive Domain: Knowing	Content Domain: Coordinate geometry

24. If the roots of the quadratic equation $x^2 - 5x + 6 = 0$ are the intercepts of a line, then find the equation of the line.

a. $2x + 3y = 6$
b. $3x + 2y + 6$
c. Either (a) or (b)
d. neither (a) nor (b)

Cognitive Domain: Reasoning	Content Domain: Coordinate Geometry

25. Find the equation of median drawn to the side BC of ΔABC whose vertices are A(1, –2), B(3, 6) and C(5, 0).

a. $3x – 5y – 11 = 0$
b. $3x – 5y + 11 = 0$
c. $5x + 3y – 11 = 0$
d. $5x – 3y – 11 = 0$

Cognitive Domain: Knowing	Content Domain: Coordinate Geometry

26. Find the area of the triangle formed by the line $3x - 4y + 12 = 0$ with the coordinate axes.

a. 6 sq. units
b. 12 sq. units
c. 1 sq. units
d. 36 sq. units

Cognitive Domain: Applying	Content Domain: Statistics

27. If the arithmetic mean of the following distribution is 8.2, then find the value of p.

x	1	3	5	9	11	13
f	3	2	7	p	4	8

a. 5
b. 6
c. 9
d. 12

Cognitive Domain: Reasoning	Content Domain: Statistics

28. The mean of the following distribution is 56, but the frequencies f_1 and f_2 in classes 20 - 40 and 80 - 100 respectively are missing. Find the missing frequencies.

Class - interval	Frequency
0 - 20	16
20 - 40	f_1
40 - 60	25
60 - 80	16
80 - 100	f_2
100 -120	10
Total	90

a. $f_1 = 12, f_2 = 13$
b. $f_1 = 13, f_2 = 12$
c. $f_1 = 11, f_2 = 12$
d. $f_1 = 12, f_2 = 11$

Cognitive Domain: Applying	Content Domain: Statistics

29. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is a two-digit prime number?

a. 1/18
b. 5/12
c. 1/6
d. 1/36

Cognitive Domain: Reasoning	Content Domain: Statistics

30. x = ABCDEFGH...Z. Find the probability of a letter selected from those in odd positions of x being a vowel.

a. 6/13
b. 5/13
c. 7/13
d. 4/13

Scholar Section (Each Question is 2 Marks)

Cognitive Domain: Reasoning	Content Domain: Number system

31. If ab + 4 = cd and ba + 40 = dc, where ab, cd, ba and dc are 2-digit prime numbers. Further, b and d are the prime numbers and a, c are neither prime nor composite. Find the value of (ab + ba)/(cd + dc).

a. 1
b. 1/8
c. 1/4
d. 1/2

Cognitive Domain: Applying	Content Domain: Algebra

32. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a – b, a, a + b, then find the value of a and b.

a. $1, 2\sqrt{2}$
b. $\pm\sqrt{2}, 2$
c. $1, \pm\sqrt{2}$
d. $-\sqrt{2}, 1$

Cognitive Domain: Knowing	Content Domain: Algebra

33. Find the value of x and y.

$$\frac{2x+1}{3} + \frac{3y+2}{5} = 2 \; and \; \frac{2(2x+1)}{3} - \frac{3(3y+2)}{5} = -1$$

a. x = 1, y = 1
b. x = -1, y = 1
c. x = 1, y = -1
d. x = 1/2, y = 1

Cognitive Domain: Reasoning	Content Domain: Algebra

34. Eighteen men can complete work in 14 days. Three women do as much work as two men. Five men and six women started the work and continued for 4 days. Subsequently, 3 more men joined the group. In how many total days was the work completed?

a. 17⅓
b. 21⅓
c. 22
d. 18

Cognitive Domain: Reasoning	Content Domain: Algebra

35. Find the sum of the series 6 + 66 + 666 + ……. upto n terms.

a. $\frac{2(10^{n+1} - 8n - 10)}{27}$

b. $\frac{2(10^{n+1} - 9n - 9)}{27}$

c. $\frac{2(10^{n+1} - 10n - 9)}{27}$

d. $\frac{2(10^{n+1} - 9n - 10)}{27}$

Cognitive Domain: Applying	Content Domain: Trigonometry

36. Find the value of $(2x)/(1 - x^2)$ if the value of x is given as below:

$$x = \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

a. cos θ
b. sin θ
c. cot θ
d. tan θ

Cognitive Domain: Reasoning	Content Domain: Trigonometry

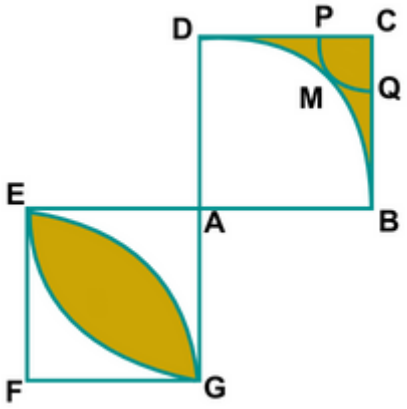
37. Find the value of tan x.cot y, if:

$$\frac{\text{Sin}(x - y)}{\text{Sin}(x + y)} = \frac{3}{5}$$

a. 4
b. 3
c. 2
d. 1

Cognitive Domain: Reasoning	Content Domain: Mensuration

38. ABCD and EFGA are the squares of side 4 cm each. In square ABCD, DMB and PMQ are the arcs of circles with centres at A and C respectively. In square AEFG, the shaded region is enclosed by two arcs of circles with centres at A and F respectively. What is the ratio of the shaded regions of the squares ABCD and AEFG respectively?



a. $\frac{2 + \pi (2 - \sqrt{2})}{\pi - 2}$

b. $\frac{2 + \pi (\sqrt{2} - 2)}{\pi - 2}$

c. $\frac{(\pi - 2)}{2 (\sqrt{2} + 1 - \pi)}$

d. $\frac{\sqrt{2} + \pi (2 - \sqrt{2})}{\pi - 2}$

Cognitive Domain: Knowing	Content Domain: Statistics

39. If a two-digit number is chosen at random, then find the probability that the number chosen is a multiple of 3.

a. 7/25
b. 29/100
c. 3/10
d. 1/3

Directions (40-42): Study the passage given below and answer the questions based on it.

The students in Mrs Thompson's math class are practising arithmetic progressions (APs), which are sequences of numbers in which the difference between consecutive terms is constant. One day, she challenges the class with a practical application of APs.
Mrs. Thompson explains, "Imagine you are creating a staircase where each step up represents an increase in the sequence. Your starting step is 2 units high, and each subsequent step goes up by 3 units more than the previous one. We'll explore how high the staircase goes over several steps and what patterns we observe."
She sets out the details: the first term of the AP (the height of the first step) is 2, the common difference (the increase in height from one step to the next) is 3, and the students are to calculate heights and sum up to different terms.

Cognitive Domain: Applying	Content Domain: Algebra

40. If the pattern of increase changes after the 10th step, with each subsequent step increasing by 5 units instead of 3, what is the height of the 20th step?

a. 70
b. 89
c. 79
d. 129

Cognitive Domain: Applying	Content Domain: Algebra

41. If the students calculate the total height of the staircase up to the 10th step, what would be the sum?

a. 155
b. 160
c. 170
d. 165

Cognitive Domain: Reasoning	Content Domain: Algebra

42. Which term of this arithmetic progression will be the first to exceed a height of 50 units?

a. 11th term
b. 15th term
c. 17th term
d. 18th term

Directions (43-45): Carefully read through the passage and answer the following questions.

Science Project

David, a 10th-grade student, makes a project on coronavirus in science for an exhibition at his school. In this project, he picks a sphere which has a volume of 38808 cm³ and 11 cylindrical shapes, each with a volume of 1540 cm³ and a length of 10 cm.

Cognitive Domain: Knowing	Content Domain: Mensuration

43. Find the diameter of the base of the cylinder.

a. 7 cm
b. 14 cm
c. 16 cm
d. 12 cm

Cognitive Domain: Applying	Content Domain: Mensuration

44. Find the volume of the shape formed.

a. 45738 cm³
b. 85541 cm³
c. 55748 cm³
d. 24625 cm³

Cognitive Domain: Applying	Content Domain: Mensuration

45. Find the volume of the shape formed. Find the total area covered by cylindrical shapes on the surface of sphere. (in approx.)

a. 1470 cm^2
b. 1896 cm^2
c. 1580 cm^2
d. 1694 cm^2

Answer Key

1.	a	**2.**	b	**3.**	c	**4.**	a	**5.**	b	**6.**	d	**7.**	b
8.	c	**9.**	a	**10.**	b	**11.**	c	**12.**	d	**13.**	a	**14.**	c
15.	b	**16.**	c	**17.**	a	**18.**	b	**19.**	c	**20.**	b	**21.**	c
22.	d	**23.**	a	**24.**	c	**25.**	d	**26.**	a	**27.**	b	**28.**	c
29.	a	**30.**	b	**31.**	d	**32.**	c	**33.**	a	**34.**	c	**35.**	d
36.	d	**37.**	a	**38.**	b	**39.**	d	**40.**	c	**41.**	a	**42.**	d
43.	b	**44.**	c	**45.**	d								